Fontys Hogescholen Engineering

Mathematics – Substitution and Partial Integration

Example Answer Sheet Exercise 9 & 10

Exercise 9b

$$\int_0^{\frac{5}{6}\pi} \cos(t)\, sin^2(t)\, dt$$

$$\text{y} = \sin(t)\,, dy = \cos(t)\, dt, \sin(0) = 0, \sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$$

$$\int_0^{\frac{1}{2}} y^2 dy = \left[\frac{1}{3}y^3\right]_{y=0}^{\frac{1}{2}} = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{3}(0)^3 = \frac{1}{24} - 0 = \frac{1}{24}$$

Exercise 9d

$$\int \frac{2t}{cos^2(t^2+4)} dt$$

$$y = t^2 + 4, dy = 2tdt$$

$$\int \frac{1}{cos^2(y)} dy = \tan(y) + C = \tan(t^2+4) + C$$

Exercise 10a

$$\int x \sin(x)\, dx$$

$$f(x) = x \to f'(x) = 1$$
$$g'(x) = \sin(x) \to g(x) = -\cos(x)$$

$$= x \cdot \cos(x) - \int 1 \cdot - \cos(x)\, dx$$
$$= -x\cos(x) - \int - \cos(x)\, dx$$
$$= -x\cos(x) + \int \cos(x)\, dx$$
$$= -x\cos(x) + \sin(x) + C$$

Exercise 10f

$$\int_1^2 (x+3)^2 e^x \, dx$$

$$f(x) = (x+3)^2 \rightarrow f'(x) = 2(x+3)$$
$$g'(x) = e^x \rightarrow g(x) = e^x$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - \int_1^2 e^x \cdot 2(x+3) \, dx$$

$$f(x) = 2(x+3) \rightarrow f'(x) = 2$$
$$g'(x) = e^x \rightarrow g(x) = e^x$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - \left([2(x+3) \cdot e^x]_{x=1}^2 - \int_1^2 2e^x \, dx \right)$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - [2(x+3) \cdot e^x]_{x=1}^2 + [2e^x]_{x=1}^2$$

$$= (5^2 \cdot e^2 - 4^2 \cdot e^2) - (2(5)e^2 - 2(4)e) + (2e^2 - 2e)$$

$$= 25e^2 - 16e - 10e^2 + 8e + 2e^2 - 21e = 17e^2 - 10e$$