

## Fontys Hogescholen Werktuigbouwkunde

### Wiskunde – Substitutieregel en Partieel Integreren

#### Uitwerkingen Opdracht 9 & 10

##### Opdracht 9b

$$\int_0^{\frac{5\pi}{6}} \cos(t) \sin^2(t) dt$$

$$y = \sin(t), dy = \cos(t) dt, \sin(0) = 0, \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\int_0^{\frac{1}{2}} y^2 dy = \left[ \frac{1}{3} y^3 \right]_{y=0}^{\frac{1}{2}} = \frac{1}{3} \left( \frac{1}{2} \right)^3 - \frac{1}{3} (0)^3 = \frac{1}{24} - 0 = \frac{1}{24}$$

##### Opdracht 9d

$$\int \frac{2t}{\cos^2(t^2 + 4)} dt$$

$$y = t^2 + 4, dy = 2t dt$$

$$\int \frac{1}{\cos^2(y)} dy = \tan(y) + C = \tan(t^2 + 4) + C$$

##### Opdracht 10a

$$\int x \sin(x) dx$$

$$f(x) = x \rightarrow f'(x) = 1$$

$$g'(x) = \sin(x) \rightarrow g(x) = -\cos(x)$$

$$= x \cdot \cos(x) - \int 1 \cdot -\cos(x) dx$$

$$= -x \cos(x) - \int -\cos(x) dx$$

$$= -x \cos(x) + \int \cos(x) dx$$

$$= -x \cos(x) + \sin(x) + C$$

**Opdracht 10f**

$$\int_1^2 (x+3)^2 e^x dx$$

$$f(x) = (x+3)^2 \rightarrow f'(x) = 2(x+3)$$

$$g'(x) = e^x \rightarrow g(x) = e^x$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - \int_1^2 e^x \cdot 2(x+3) dx$$

$$f(x) = 2(x+3) \rightarrow f'(x) = 2$$

$$g'(x) = e^x \rightarrow g(x) = e^x$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - \left( [2(x+3) \cdot e^x]_{x=1}^2 - \int_1^2 2e^x dx \right)$$

$$= [(x+3)^2 \cdot e^x]_{x=1}^2 - [2(x+3) \cdot e^x]_{x=1}^2 + [2e^x]_{x=1}^2$$

$$= (5^2 \cdot e^2 - 4^2 \cdot e^2) - (2(5)e^2 - 2(4)e) + (2e^2 - 2e)$$

$$= 25e^2 - 16e^2 - 10e^2 + 8e + 2e^2 - 21e = 17e^2 - 10e$$